12th International Conference on Hydroscience & Engineering
Hydro-Science & Engineering for Environmental Resilience
November 6-10, 2016, Tainan, Taiwan.

Statistical Characteristics of Bed Load Rolling

Ying Shen[1], Hai-Jue Xu[2], Sheng-Fa Yang[1]
1. National Engineering Research Center for Inland Waterway Regulation, Chongqing Jiaotong University
Chongqing, China
2. State Key Laboratory of Hydraulic Engineering Simulation and Safety, Tianjin University
Tianjin, China

ABSTRACT

A theoretical research on characteristics of bed load rolling is presented in this paper. For bed load particles, rolling is the major transport mode at low flow intensity. Therefore, the forces and moments acting on a single particle are analyzed, and a theoretical model of bed load rolling that accounts for the role of initial grain position is established. The expressions for the rate and distance of a single-step movement are obtained by numerical calculation and regression analysis. As a result, an improved Einstein's bed load function for the lower Shields number is derived and verified, which performs better than Einstein's.

KEY WORDS: gravel particles; bedload transport rate; bedload rolling

INTRODUCTION

The transport characteristics of the gravel bedload have raised increasing concern from Chinese sediment researchers in recent few decades. The reason is that after many large water conservancy projects had been built in southwest China, the considerable amount of the gravel bedload transport is problematic (Zhang, 2005). Hence, it is of great significance to study the characteristics of bedload rolling in order to develop the mountainous rivers.

Generally, bedload may roll, slide or saltate, depending on flow and bed conditions. For low-flow intensity, rolling is the major mode, making up more than 80% of the total bedload transport for Shields number $\Theta < 0.08$ (Hui, 1991).

Therefore, the objective of this research is first to improve the understanding of bedload rolling on the bottoms of rivers, but also to supply a basis on the prediction on river migration.

THEORETICAL MODEL OF BEDLOAD ROLLING

Physical Phenomena and Simplification of Particle Rolling

The rolling processes are complicated, because the velocity of turbulent flow is fluctuating, the positions of particles on bed are random and the interaction between water and sediment is active and frequent. However, the processes can still be decomposed as a series of single steps. Then, the processes of the particles are simplified as Fig. 1.

Referring to Fig. 1, A is the moving particle on bed, while particle B is fixed on bed. Particle A rolls over the head of B under the action of turbulent flow near bed. Particle A may leaps over particle B at certain point due to the centrifugal force which varies with the relative position of A to B and the flow velocity. The greater the flow velocity, the more possible that A departs B before the vertex of particle B. Thus, a single step is divided into the contact step and the free step (Fig. 1). Moreover, the contact step is separated from the free step in this research with the standard for the judgment being whether the contact force greater than zero.

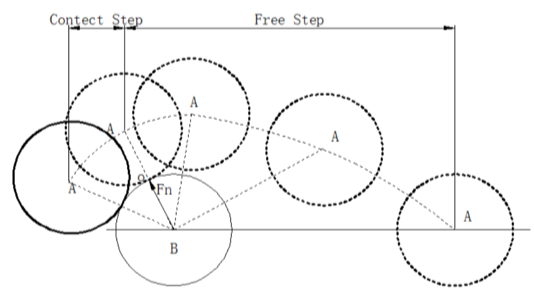

Fig. 1 Schematic diagram of bedload rolling

Formulation

Dynamic equations for contact step. Referring to Fig. 2, when the water flows through the loose riverbed, the forces acting on the particle include the drag F_D, lift F_L, added mass force F_m and contact force F_n. According to the principle of force balance, the equation of particle A along n direction is:

$$m_s D\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 = F_{Dx}\cos\alpha - F_{Dy}\sin\alpha + F_{Lx}\cos\alpha - F_{Ly}\sin\alpha + W'\sin\alpha - F_n - F_{mn} - F_{Bn} \tag{1}$$

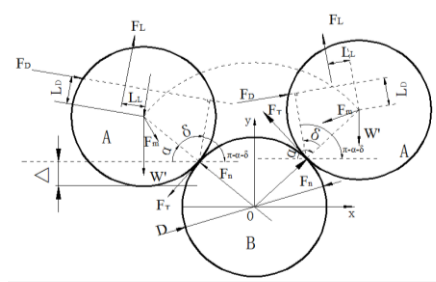

Fig. 2 Schematic diagram for contact step of particle rolling

where $m_s = \rho_s A_3 D^3$ is the particle mass with ρ_s as the sediment density, A_3 as the correction coefficient of sediment volume and D as the particle's diameter; α is the angle between F_n and the direction of x negative; $F_D = C_D A_1 D^2 \rho[(u_b-u)^2+v^2]/2$ is the drag force with C_D as the drag coefficient, A_1 as the correction coefficient of sediment area for the drag force, ρ as the density of water, u_b as the flow velocity near bed, u and v respectively as the longitudinal and vertical component of the particle rolling velocity, $F_{Dx} = C_D A_1 D^2 \rho[(u_b-u)^2+v^2]^{1/2}(u_b-u)/2$ as the longitudinal component and $F_{Dy} = C_D A_1 D^2 \rho[(u_b-u)^2+v^2]^{1/2}(-v)/2$ as the vertical component of drag force vector; $F_L = C_L A_2 D^2 \rho[(u_b-u)^2+v^2]/2$ is the lift force with C_L as the lift coefficient, A_2 as the correction coefficient of sediment area for the lift force, $F_{Lx} = C_L A_2 D^2 \rho[(u_b-u)^2+v^2]^{1/2} v/2$ as the longitudinal component and $F_{Ly} = C_L A_2 D^2 \rho[(u_b-u)^2+v^2]^{1/2}(u_b-u)/2$ as the vertical component of lift force vector; $W' = A_3(\rho_s-\rho)gD^3$ is the submerged gravity with g as the gravitational acceleration; $\boldsymbol{F}$ is the contact force vector from particle B with F_t, F_n as the tangential and the normal components of the contact force, respectively; $\boldsymbol{F}_m = -C_{A0} m_f D \times \mathrm{d}\boldsymbol{u}/\mathrm{d}t$ is the added mass force vector with C_{A0} as its coefficient, $m_f = \rho A_3 D^3$ as the mass of water, $\boldsymbol{u}$ as the velocity vector of sediment and F_{mt}, F_{mn} as the tangential and the normal components of the added mass force vector, respectively; $\boldsymbol{F}_B$ is the Basset force vector with F_{Bt} and F_{Bn} as the tangential and the normal components respectively; L_D is the vertical distance from center of A to locus of F_D; L_L is the horizontal distance from the center of A to locus of F_L; Δ is the exposure degree, which is a random variable; (x, y) are the horizontal and vertical axis of Cartesian coordinates.

The Basset force was proposed by Basset (1888, 1910) and is caused by the unsteady viscous shear on the particle surface (Basset, 1910). Guo (2011) stated the original form of the Basset force is suitable for the flow with small Reynolds numbers; as to that with large Reynolds numbers, it can be combined with the added mass force, and rewritten as an integrated form,

$$C_{A0} m_f D\left(\frac{d\alpha}{dt}\right)^2 + F_{Bn} = C_A m_f D\left(\frac{d\alpha}{dt}\right)^2 \tag{2}$$

where C_A is the integrated added mass coefficient.

In Eq. 1, there are 2 unknowns, therefore, we still need another equation, the moment equilibrium equation of particle A around instantaneous center (contact point of A and B) (Han and He, 1984),

$$J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = F_D\left(L_D + R\cos\delta\right) + F_L\left(L_L + R\sin\delta\right) - W'R\cos\alpha \tag{3}$$

where $J = 7(m_s+m')R^2/5$ is the inertia moment of particle A to instantaneous center with $R = D/2$ as the radius of sediment particle and $m' = C_A A_3 \rho D^3$ as the equivalent mass of the added mass force and Basset force; $\theta = 2\alpha$ is the angle that particle A rotates around centroid of A; $\delta = \arccos\{[(u_b-u)\times\tan\alpha - v]\times\cos\alpha/[(u_b-u)^2+v^2]^{1/2}\}$ is the angle between F_n and the perpendicular of F_D.

Substituting the expressions of m_s, m', θ, J, F_D, F_L and W' into Eq. 3 yields,

$$\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} = \frac{5}{7}\frac{1}{\frac{\rho_s}{\rho}+C_A}\left\{\left[\left(\frac{u_b}{D}-\frac{\mathrm{d}\alpha}{\mathrm{d}t}\sin\alpha\right)^2+\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\cos\alpha\right)^2\right]\left[\frac{A_1C_D}{2A_3}\left(\frac{L_D}{R}+\cos\delta\right)\right.\right.$$
$$\left.\left.+\frac{A_2C_L}{2A_3}\left(\frac{L_L}{R}+\sin\delta\right)\right]-\left(\frac{\rho_s}{\rho}-1\right)g\frac{\cos\alpha}{D}\right\} \tag{5}$$

If it is assumed that the sediment starts from rest, then the initial conditions for Eqs. (2) and (5) is $\mathrm{d}\alpha/\mathrm{d}t\mid_{t=0} = 0$, $\alpha\mid_{t=0} = \alpha_0$, where α_0 is the initial angle between F_n and the minus direction of x axis, which is closely associated with the exposure degree Δ.

Determination of parameters. For simplicity, sphere is taken as the approximate shape of a sediment particle. All the parameters and their corresponding values are listed in Table 1. The most intense debate exists in the determination of the drag coefficient C_D and the lift coefficient C_L. For simplicity, many investigators (Han, 1984) adopted $C_D \approx 0.4$ and $C_L \approx 0.1$, proposed by ДеМентъев. However, there is a problem with these parameters, which are determined from the data of the experiment done on a cylinder, but not on a sphere or a real sediment particle. Therefore, the mismatch of the parameters could causes a conflict between the model and the parameters which will be discussed in detail below. The research adopts $C_D = 0.7$ and $C_L = 0.18$ which were from the experiment done by Li (1983) on single particle on smooth bed.

Table 1. Parameters and corresponding values in the equations

Name of the Parameters	Symbol	Value
area coefficient of drag force	A_1	π/4
area coefficient of lift force	A_2	π/4
area coefficient of submerged gravity	A_3	π/6
drag coefficient	C_D	0.7
lift coefficient	C_L	0.18
coefficient of vertical motion resistance	C	1.2
arm of drag force	L_D	D/6
arm of lifting force	L_L	0

Critical condition for separation. The critical condition for the separation is that the normal contact force F_n equals zero, that is, when $\alpha = \beta, F_n = 0$, where β is the separation angle. Thus, Eq. 1 is rewritten as,

$$-\frac{3}{4}\left[\left(\frac{u_b}{D}-\frac{\mathrm{d}\alpha}{\mathrm{d}t}\sin\alpha\right)^2+\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\cos\alpha\right)^2\right]^{\frac{1}{2}}\left(C_D\frac{u_b}{D}\cos\alpha + C_L\frac{u_b}{D}\sin\alpha + C_L\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)\Bigg|_{\alpha=\beta}$$
$$+\left(\frac{\rho_s}{\rho}+C_A\right)\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2-\left(\frac{\rho_s}{\rho}-1\right)\frac{g}{D}\sin\alpha\Bigg|_{\alpha=\beta}=0 \tag{5}$$

During the integration, when Eq. 5 is satisfied, then the calculation on the contact step finishes, and that on the free step starts.

Equations for free step. After departing from particle B, particle A does not bear the counterforce any more, and the trajectory is no longer a circle, but is determined by the movement itself. The details are shown in Fig. 3.

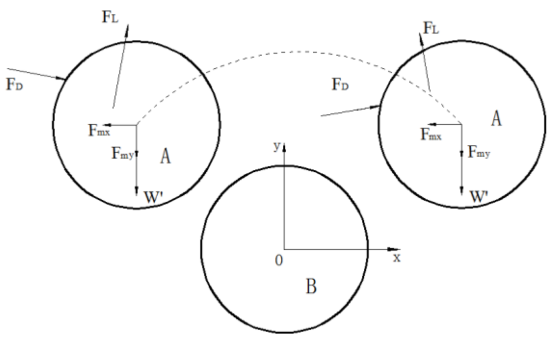

Fig. 3 Schematic diagram for free step of particle rolling

According to the principle of the force balance, the equations can be expressed as follows:

$$m_s\frac{\mathrm{d}u}{\mathrm{d}t}=F_{Dx}+F_{Lx}-F_{mx} \tag{6}$$

$$m_s\frac{\mathrm{d}v}{\mathrm{d}t}=F_{Dy}+F_{Ly}-W'-F_{my} \tag{7}$$

where F_{mx} and F_{my} are the horizontal and the vertical components of the added mass force and Basset force, respectively.

Substituting the expressions of F_{Dx}, F_{Dy}, F_{Lx}, F_{Ly} and W' into Eqs. 6 ~ 7 and noting that u = dx/dt, v = dy/dt, we get,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{C_DA_1}{2(\rho_s/\rho+C_A)A_3D}\left[\left(u_b-\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right]^{\frac{1}{2}}\left(u_b-\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$
$$+\frac{C_LA_2}{2(\rho_s/\rho+C_A)A_3D}\left[\left(u_b-\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right]^{\frac{1}{2}}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) \tag{8}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=\frac{1}{2(\rho_s/\rho+C_A)}\left\{\frac{C_DA_1}{A_3D}\left[\left(u_b-\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right]^{\frac{1}{2}}\left(-\frac{\mathrm{d}y}{\mathrm{d}t}\right)+\right.$$
$$\left.\left(\frac{C_LA_2}{A_3D}-\frac{2}{\rho_s/\rho+C_A}\right)\left[\left(u_b-\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2\right]^{\frac{1}{2}}\left(u_b-\frac{\mathrm{d}y}{\mathrm{d}t}\right)\left(\frac{\rho_s}{\rho}-1\right)g\right\} \tag{9}$$

INCIPIENT SHIELDS NUMBER

When sediment particle is about to move, $\alpha=\alpha_0$, $\mathrm{d}^2\alpha/\mathrm{d}t^2=0$, $\mathrm{d}\alpha/\mathrm{d}t=0$. Substituting the values of A_1, A_2, A_3, C_D, C_L, L_D , L_L and noticing that $\sin\alpha_0 = 1-\Delta'$, $\cos\alpha_0 = (2\Delta'-\Delta'^2)^{-1/2}$ and $\delta_0 = \pi/2-\alpha_0$, into Eq. 4 ,we get

$$u_{b,1}=\frac{\sqrt{\frac{4}{3}\left(\frac{\rho_s}{\rho}-1\right)gD}}{\sqrt{0.7\frac{\frac{4}{3}-\Delta'}{\sqrt{2\Delta'-\Delta'^2}}+0.18}} \tag{10}$$

where $\Delta' = \Delta/R$ is the relative exposure degree, with its distribution (Han and He, 1984) as the following form

$$f(\Delta')=\begin{cases}1, & 0<\Delta'\le 1\\ 0, & others\end{cases} \tag{11}$$

Then the mathematical expectation of incipient velocity $\bar{u}_{b,1}$ can be obtained as follows,

$$\overline{u_{b,1}}=\int_0^1 u_{b,1}f(\Delta')\mathrm{d}\Delta'=1.231\sqrt{(\rho_s/\rho-1)gD} \tag{12}$$

where $\bar{u}_{b,1}$ is the mathematical expectation of incipient velocity.
If the logarithmic velocity distribution is adopted and the particle is in hydraulic roughness region, we can figure out

$$u_b=5.77u_* \tag{13}$$

Substituting Eq. 12 into Eq. 13 and considering the Shields number $\Theta = u_*^2/((\rho_s/\rho-1)\times gD)$, it is obtained that the incipient Shields number Θ_c equals 0.0455, which is close to 0.045 given by Miller *et al*. (1977). Besides, Chien (1986) also stated that the upper limit of Θ_c is 0.06, while the lower limit is about 0.04. The result given by this research is well in this range. Hence, the formulae of incipient velocity given in this research is reasonable.

ANALYSIS OF ROLLING MOTION

The fourth-order Runge-kutta method and the regression method is used to analyze the averaged velocity L and the rolling distance U_s with Θ and Δ' as the independents. And the dimensionless expressions of single-step parameters are as follows:

$$\frac{L}{D}=10.26(\Delta')^{0.12}\Theta^{0.45} \tag{14}$$

$$\frac{U_s}{\sqrt{(\rho_s/\rho-1)gD}}=15.39(\Delta')^{-0.24}\Theta^{1.3} \tag{15}$$

The mathematical expectations of the parameters can be obtained from integrating Eqs. 14 and 15, respectively,

$$\frac{\bar{L}}{D}=\int_0^1\frac{L}{D}f(\Delta')\mathrm{d}\Delta'=9.16\Theta^{0.45} \tag{16}$$

$$\frac{\bar{U}_s}{\sqrt{(\rho_s/\rho-1)gD}}=20.25\Theta^{1.3} \tag{17}$$

where $\bar{L}$ is the mathematical expectation of the single-step distance; $\bar{U}_s$ is the mathematical expectation of single-step velocity.

SEDIMENT TRANSPORT RATE OF ROLLING BEDLOAD

Bedload transport formula is used to calculate the transport rate when the sediment exchanging between the riverbed and the bedload region achieves its equilibrium state under certain flow conditions. At that time, the silting equals the picking-up amount of the particles (Chien, 1986). The transport rate is the weight of the sediment passing through a unit width in a unit time.

According to Einstein's (1950) theory on bedload movement,

$$P_S = \frac{g_b(1-p)}{\bar{L}} \tag{18}$$

$$P_p = \frac{\alpha_2}{\alpha_1\alpha_3} \cdot \gamma_s p \sqrt{\frac{\rho_s - \rho}{\rho} gD} \tag{19}$$

where P_s is the deposition rate; g_b is the bedload transport rate calculated in dry weight; p is the erosion probability; P_p is the erosion rate; α_1, α_2 is the constant related to shape of sediment particle; α_3 is the constant related to flow resistance coefficient.

Einstein stated that the erosion probability p is determined by of sediment transport characteristics and flow pattern near bed. When the lift force acted on the particle is greater than the submerged weight of it, then the particle begins to move. Therefore the erosion probability equals the probability of the lift force greater than the particle submerged weight, which means $p = p\ (F_L/W' > 1)$. However, many investigators found that the sediment incipient motion is not only related to the lift force, but also related to the drag force (Wu and Lin, 2002). After Wu and Lin (2002) had analyzed the experiment data supplied by Guy et al.(1966), Luque (1974), Jain (1992)and Papanicolaou (1999), they then obtained the formula of pick-up probability of the sediment as follows:

$$p = 0.5 - 0.5\frac{\ln(0.044/\Theta C_L)}{|\ln(0.044/\Theta C_L)|}\sqrt{1 - \mathrm{e}^{-\frac{2}{\pi}\left(\frac{\ln\left(\frac{0.044}{\Theta C_L}\right)}{0.724}\right)^2}} \tag{20}$$

if C_L = 0.18 is considered, then p only depends on Shield number. If the sediment exchanging achieves the equilibrium state, then,

$$\frac{g_b(1-p)}{\bar{L}} = \frac{\alpha_2}{\alpha_1\alpha_3} \cdot p\gamma_s \sqrt{\frac{\rho_s - \rho}{\rho} gD} \tag{21}$$

which could also be written as,

$$\Phi = A_* \frac{\bar{L}}{D}\frac{p}{1-p} \tag{22}$$

where A* = $\alpha_2/(\alpha_1 \times \alpha_3)$ = 0.023 (Meyer-Peter et al., 1948); $\Phi = g_b/(\gamma_s D((\rho_s-\rho)/\rho g D)^{1/2})$ is the dimensionless transport rate.

Table 2 lists the formulae of the bedload rolling transport rate given by this research, Han and He (1999) and Einstein, respectively. Fig. 5 shows the calculation results of these formulae and the experimental data which were obtained from the flume experiment of the gravel done by Meyer-Peter et al. (1948).

Table 2. Formulae of uniform sediment transport rate

Author	Formula	Applicable condition
This research	$\Phi = A_* \frac{\bar{L}}{D}\frac{p}{1-p}$	$\Theta < 0.1$
Han and He	$\Phi = \frac{2}{3} m_0 \varepsilon_1 \frac{U_2}{\sqrt{(\rho_s/\rho - 1)gD}}$	$\Phi < 3.65 \times 10^{-3}$
Einstain	$\frac{A_*\Phi}{1+A_*\Phi} = 1 - \frac{1}{\sqrt{\pi}}\int_{-B_*\psi-\frac{1}{\eta_0}}^{B_*\psi-\frac{1}{\eta_0}} e^{-t^2} dt$	

Where m_0 is the area compact coefficient of surface particles; ε_1 is the incipient probability of sediment; U_2 is the averaged rolling velocity of particles; B* = 1/7; Ψ = 1/Θ; η_0 = 1/2.

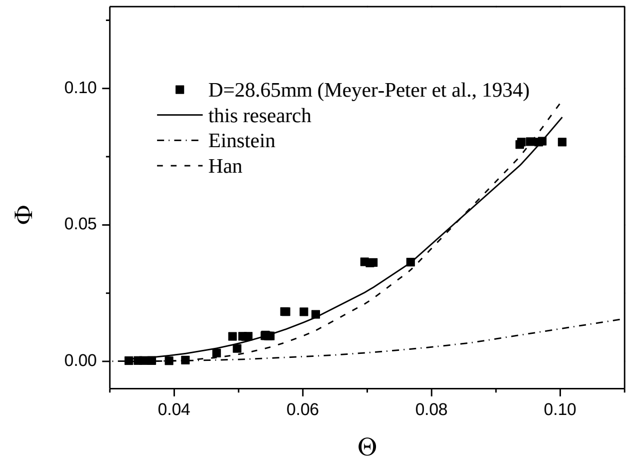

Fig. 5 Comparison between calculation results of transport rate ormulae and experimental data

Fig. 5 shows that when Θ < 0.1, the results of the formula given in this research fit well with the experimental data. When Shields number Θ > 0.1, the percentage of the saltation load increases sharply with the continuous increase of Shields number. Therefore, the model used herein could not be used to predict the bedload transport rate when Θ > 0.1.Besides, the overall results given by Einstein (1950) agree with the experimental data, however some in the range of 0.04 < Θ < 0.08 is a little bit smaller than expected. The results given by Han and He (1999) only seems good in the range of Θ < 0.042, but those in the range of Θ > 0.042 is much smaller than the observed, which were probably due to that the different values of C_D and C_L used by Han and He from this study.

CONCLUSIONS

After solving the mathematic model on the rolling process of a sediment particle, the expression of the averaged rolling distance and the rolling velocity are obtained by using the Runge-Kutta method and the regression analysis. The bedload transport formula is derived by combining the probability theory and the mechanical principle. And the calcution results are compared with the experimental data to prove the validity of the model. Some conclusions are reached as follows: the critical dimensionless shear stress for the incipient motion of the sediment is 0.0455; the characteristic parameters of particle, such as the rolling distance are not only depended on Shields number but also depended on their relative exposure degree which is therefore treated as an important factor in this research; the proposed formula fit well with the measured data .

REFERENCE

Basset, A. B. (1888). A Treatise in Hydrodynamics, Deighton Bell. Cambradge UK.

Basset, A. B. (1910). Descent of a sphere in a viscous fluid. Quarterly Journal of Mathematics, Vol. 41, pp. 369-381.

Chien, N. and Wan Z. H. (1986). Mechanics of Sediment Transport (in Chinese), Science Press, Beijing.

Einstein, H. A. (1950). The Bed Load Function for Sediment Transportation in Open Channel Flows. Technical Bulletin, United States Department of Agriculture Soil Conservation Service, No. 1026, Washington.

Fahnestoch, R. K. (1963). Morphology and Hydrology of a Glacial Stream-White River, Mount Rainier, Washington, Geological Survey Professional Paper: 422-A, Washington.

Gilbert, G. K. and Murphy, E. C. (1914). The Transportation of Debris by Running Water, Usgs Professional Paper: 86, Washington.

Guo, J. (2011). Motion of spheres falling through fluids. Journal of Hydraulic Research, Vol. 49, No. 1, pp. 32-41.

Guy, H. P., Simons, D. B. and Richardson, E. V. (1966). Summary of Alluvial Channel Data from Flume Experiments, 1956-61, Usgs Professional paper: 462-I, Washington.

Han, Q. W. and He, M. M. (1984). Statistical Theory of Sediment Movement (in Chinese). Science Press, Beijing.

Han, Q. W., He, M. M. (1999). Characteristics of Incipient Sediment Motion and Incipient Velocity (in Chinese). Beijing: Science Press.

Helley, E. I. (1969). Field Measurement of the Initiation of Large Bed Particle Motion in Blue Greek near Klamath California, Usgs Professional Paper: 562-G, Washington.

Hui, Y. J. and Hu, C. H. (1991). Kinematic characteristics of saltation particle in water flow (in Chinese). Journal Hydraulic Engineering, 1991, No. 4, pp. 59-64.

Jain, S. C. (1992). Note on lag in bed load discharge. Journal of Hydraulic Engineering, Vol. 118, No. 6, pp. 904-917.

Jalaal, M., Ganji, D. D. and Ahmadi, G. (2010). Analytical investigation on acceleration motion of a vertical falling spherical particle in incompressible Newtonian media. Advanced Powder Technology, Vol. 21, No. 3, pp. 298-304.

Li, Z. R., Chen, Y. E. and Zhao, Y. (1983). Laboratory investigation acting on drag and lift forces acting on bed pheres(in Chinese). Proceedings of the Second International Symposium on River Sedimentation, Water Resources and Electric Power Press, Beijing, No. 10, pp. 330-343.

Miller, M. C., McCave, I. N. and Komar, P. D. (1977). Threshold of sediment motion under unidirectional currents. Sedimentology, Vol. 24, No. 4, pp. 507-527.

Shields, A. (1936). Anwendung der aehnlichkeitsmechanik und der turbulenzforschung auf die Geschiebebewegung, Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau, Berlin, No. 26.

Tang, L. M. and Wang, X. K. (2009). Statistical analysis on motion characteristics of saltating particle(in Chinese). Journal of Hohai University (Nature Science), No. 2, pp. 232-236.

Wu, F. C. and Lin, Y. C. (2002). Pickup probability of sediment under log-normal velocity distribution. Journal of Hydraulic Engineering, Vol. 128, No. 4, pp. 438-442.

Yang, F. G., Liu, X. N., Huang, E., et al. (2009). Study on the incipient velocity of Tangjiashan Barrier Lake downstream area sediment(in Chinese). Journal of Sichuan University, No. 5, pp. 84-89.

Zhang, Z. X. (2005). Study on stochastic nature of gravel bedload(in Chinese). Dissertation for the Doctoral Degree. Sichuan University, Chengdu.