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Modeling Turbidity Currents Using the Multiple-state Discrete-time Markov Chain Approach

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ABSTRACT

Turbidity currents are likely to result from flow with high sediment concentration flow into reservoirs in extreme events. The turbidity-currents deposit is an issue for reservoir desilting. This paper employs the multi-state discrete-time Markov chain approach to stimulate the turbidity flow. Markov chain can describe the random and memoryless movement of particles. The control volume is divided into various segments along the streamwise direction in this proposed model. Transport process with each segment is simulated by multi-state discrete-time Markov chain. In each state, the transition probabilities in the suspended load is determined by instantaneous fluctuating vertical velocity versus terminal settling velocity and in the bed load is determined by sediment entrainment. Moreover, adjacent segments are modeled by the birth-death process. Birth-death process imitates the sediment transition rate along the streamline and is controlled by the velocity profile. Therefore, the model is able to quantify the sediment concentration at different time and position. Model results are validated against experimental data. It is expected that this model can develop the deposits of turbidity currents in different slope and water depth considering stochastic process.

KEY WORDS: Discrete time Markov chain; sediment transport; particle tracking method; stochastic modeling; suspended sediment concentration.

INTRODUCTION

In Taiwan, the reservoirs lifespans are an important issue. According to the Water Resources Agency, Deposition volume of Zengwun Reservoir has been reach thirty-seven percent of total capacity. In the past thirty years, nine typhoons cause 75% of total Siltation. Our study provides this stochastic model with more theoretical foundation to conquer the problem and focus on the extreme event. In extreme event, the sediment under rapid current can be seen as suspended load dominant. This study will focus on the suspended load using the Multiple-state Discrete-time MC Approach. The Multiple-state Discrete-time MC Approach is based on the stochastic particle tracking which provided by Man and Tsai (2007). With the drift term and diffusion term, the particle trajectory is described as a stochastic process. The equation proposed by Man and Tsai (2007) provides the probability of the particle motion and is applied in the Multiple-state Discrete-time MC Approach. In the study, we will first introduce the MC Approach and stochastic particle tracking method. Then, the proposed model is validated against Wang and Qian (1989) with two different Rouse numbers. Finally, the constraint of the proposed model will be demonstrated.

Literature Review

The probabilistic nature of sediment incipient motion from a channel bed is a good example of stochastic hydraulics (Yen 2002). The uncertainty of sediment transport is mainly due to shape and size distribution of sediment and the turbulent near the bed. Sediment transport can be implicitly divided into two part, the entrainment on the bed load and instantaneous fluctuating velocity on the suspend load. Bed load transport results from particle rolling, sliding, and saltation. Wu and Lin (2002) provided the pickup probability using the Lognormal Velocity Distribution. The instantaneous fluctuating velocity is used to decide the particle motion. With further improvement, Wu and Yang (2004) introduced entrainment probabilities of mixed-sized sediment using the fourth-order Gram-Charlier type probability density function to describe the near-bed turbulent. In their paper, the topography of bed was considered as random process. Assuming that the exposure height and friction height are all uniformly distributed, one allows further consideration of sediment properties. Ancey (2010) used the Exner equation for planar bed to show different bed forms and combined with birth-death Markov Chain to illustrate a transport rate.

The sediment transport under turbulence can be seen as a memoryless process or Markovian properties. The motion of particle is primarily dependent on the current state (Einstein 1950; Sun and Zhu 1991; Lisle et al.1998). According to this property, Wu and Yang (2004) provided a Pseudo four-state continuous Markov Chain to calculate the number of particles and compared them with experimental data. Tsai et al. (2015) coupled a multiple-state discrete-time Markov chain to determine the suspended particle motion with fluctuating velocity. However, in their study, the layer thickness and the time step will cause some discrepancies on the result. In this paper, we will use the stochastic diffusion particle tracking model (Man and Tsai (2007)) to simulate the suspend load. The particle tracking model (PTM) has been compared with advection-diffusion (AD) equation (Man and Tsai (2007)).

METHODOLOGIES

The model can be divided into two parts, suspended load and bed load based on a reference height a=0.01*h(Van Rijn 1984). In the Multiple-state Discrete-time MC Approach, the suspended load is simulated by the particle tracking method and bed load is the fourth-order Gram-Charlier fluctuating velocity.

Multiple-state Discrete-time MC Approach

We adapt an Eulerian point of view by considering a control volume, which is divided into various segments along the streamline. Each of the segments is subdivided into equal thickness layers L. Figure 1 depicts these concepts. Each segment can be seen as a Markov Process. By the definition of the Markov chain, the probability from state i to state j can be demonstrated as

$$P\{X_{k+1} = j \mid X_k = i, X_{k-1} = i_{k-1}, ..., x_1 = i_1, X_0 = i_0\} = P_{ij} \quad (1)$$

The future state X_{k+1} is mainly relying on the current state. We define that a particle moves from layer i to layer j after one step. Layer 1 depicts the particle which is static on the bed and Layers 2 to n demonstrate the particles in different positions in the water. n is equal to water depth divided by layer thickness L.The motion of particles can be seen as stochastic process. One-step transition probabilities can be expressed by matrix P as follow.

$$P = \begin{pmatrix} P_{11} & P_{12} & K & p_{1n} \\ P_{21} & P_{22} & K & M \\ M & M & O & M \\ P_{n1} & P_{n1} & L & P_{nn} \end{pmatrix} \quad (2)$$

N_j^t vector presents the number of particles in each layer, in time step, $N_j^{t+1} = N_j^t * P$. With the initial condition, the multiple-state Discrete-time MC Approach can determine the concentration in each time step.

Transition Probability Determination (Stochastic Particle Tracking Method)
To describe the motion of suspended load, Man and Tsai (2007) provided the stochastic particle tracking method based on the Langevin equation. The governing equation includes the drift term and diffusion term.

$$dX_t = \bar{u}(t, X_t)dt + \sigma(t, X_t)dB_t \quad (3)$$

X_t represents the particle position and $\bar{u}(t, X_t)$ mean drift term can be simulated by the mean velocity and turbulent diffusion D which can be expressed as follows.

$$\bar{u}(t, X_t) = \bar{U} + \nabla D = \{ \begin{matrix} \bar{U}(x, y, z, t) + \partial D_x / \partial x \\ \bar{V}(x, y, z, t) + \partial D_y / \partial y \\ \bar{W}(x, y, z, t) - w_s + \partial D_z / \partial z \end{matrix} \quad (4)$$

$\sigma(t, X_t)dB_t$ diffusion term consists of the diffusion coefficient tensor $\sigma(t, X_t)$ and a three-dimensional vector of Wiener process dB_t . In this study, the vertical motion (direction z) of particle is described by the Stochastic Particle Tracking Method. The turbulent diffusivity D is assumed to be $k \rightarrow \infty$ where H is the water depth. Von Kármán constant (k) is about 0.4, as suggested in HÖGSTRÖM (1996). The shear velocity (u_*)is based on the flow condition. The Stochastic Particle Tracking equation based on the Euler-Maruyama method can be simplified as

$$Z_{n+1} = Z_n + (-w_s + ku_* - 2ku_* \frac{z_n}{H})Vt + (\sqrt{2ku_* z_n (1 - \frac{z_n}{H})})VB_t \quad (5)$$

$VB_t = (\sqrt{Vt}N(0,1))$ (= Gaussian-distributed random variable.
Assumed that the particle starts with the center of each layer in each time step, with the extremely small thickness layers L, this assumption can be close to nature, and the discrepancy can be ignored. By the Stochastic Particle Tracking Method, the probability of the particle moving up to the other layers can be calculated. Similarly, the particle moves down when i <0.

$$(-w_s + ku_* - 2ku_* \frac{z_n}{H})Vt + (\sqrt{2ku_* z_n (1 - \frac{z_n}{H})})(\sqrt{Vt}N(0,1)) = i*(2n-1)VL/2 \quad n = 1, 2, ..., H/L \quad i = \pm 1 \quad (6)$$

According to the equation 6, the transition probability can be estimated.

$$P_{ij} = \int_{(2(j-i)*VL/2 - (-w_s + ku_* - 2ku_* \frac{z_n}{H})Vt)/(\sqrt{2ku_* z_n (1-\frac{z_n}{H})}\sqrt{Vt}}^{(2(j-i+1)*VL/2 - (-w_s + ku_* - 2ku_* \frac{z_n}{H})Vt)/(\sqrt{2ku_* z_n (1-\frac{z_n}{H})}\sqrt{Vt}} \frac{1}{\sqrt{2\pi}} e^{-x\wedge 2} dx \quad (7)$$

On the boundary, we assume that the sediment particle will not leave the boundary. On the water surface, the moving up probability is added up and seen as the particle stays on the surface. In other words, the particle will not go through the water surface.

Birth and Death Process

To combine each segments, the Birth and Death Process is used in the study. λ_{ij}^t is defined as the arrival rate. Similarly, μ_{ij}^t is defined as leaving rate. These parameters present the sediment exchange from each segment. The first subscript (i) represents each segment; the second subscript (j) donates each layer. Nelson et al.(1995) observed entrainment is low when u<0(reverse). Proposed model ignored the motion of sediment reversing the streamline. Therefore for the adjacent segment with same layer, $\mu_{ij}^t = \lambda_{i+1,j}^{t+1}$. λ_{ij}^t is determined by the boundary. The volumetric transport rate per unit width can be expressed by

$$\mu_{ij}^{t} = N_{i,j} * V_{ij} * U_{ij} \tag{8}$$

where $V = \pi D_i^3 / 6$ is the particle volume; U is the streamline velocity; N is the particle number in each layer.

MODEL SIMULATIONS

Case Study: laboratory experiment

To consider a steady state, Wang and Qian (1989) experimented with two large discrepancy specific gravity particle(Plastic particle and beach sand). The experiment is conducted in a flume with dimensions of 20m length, 30cm width, and 40cm height. The bed slope was 1%. The channel is lined with concreter plate. Therefore, in this case, the proposed model does not consider the bed load. It is assumed that the state 1 is one class, which did not communicate with other state. $P_{1j}^{t} = 0, P_{j1}^{t} = 0$ Considering steady state, $\mu_{ij}^{t} = \lambda_{ij}^{t}$. Experiments SF2, SQ1 conducted in clean water, are validated against the proposed model.

Limiting Probabilities

According to the Chapman-Kolmogorov Equation,

$P^{(K+M)} = P^{K} * P^{M}$. When $k \to \infty$,

$\lim_{k\to\infty} \mathrm{P}^{K} = \pi_{ij}(\lim iting\ probabilities)$ which is seen as steady condition. The layer depth-average concentration can be expressed by following:

$$\overline{c_1} = \frac{\pi_1}{L_1}, \overline{c_2} = \frac{\pi_2}{L_2}, \ldots, \overline{c_n} = \frac{\pi_n}{L_n} \tag{9}$$

With the equal thickness height, $\overline{c_1} : \overline{c_2} : \ldots : \overline{c_n} = \pi_1 : \pi_{2:} : \ldots : \pi_n$.
The concentration of the particle is substituted by the limiting probabilities under steady state and then compared against experiment data.

MODEL RESULT AND DISCUSSION

Figure 2 demonstrates the comparison with experimental data and the proposed model. Reference concentration $\overline{c_a}$ is defined as the corresponding concentration at 0.16h. The time step is chosen as 0.3s; the thickness height donates 0.05*H. The discrepancy between the proposed model and experimental data is insignificant. However, the result from the Markov chain slightly underestimated the concentration. Figure3 demonstrates the condition SF2 and the simulation result under lower Rouse number which is defined as w_s / ku_*. The simulation is based on the time step =0.025s and layer thickness= 0.0025h. With lower Rouse number, the motion of particle is dominated by suspended layer. The simulation result shows some discrepancies in Figure3 compared to the Figure2. Near the bed, the concentration of the sediment particle seems to be underestimated. On the suspended load, the discrepancy can be caused from ignoring the fluctuation velocity along the streamline. It can be observed from Figure 3 that the experiment data are not so steady (smooth curve) which demonstrates the importance of the streamwise fluctuation.

Numerical stability condition of Multiple-state Discrete-time MC Approach

The MC process is a complicated implicit method. The maximum eigenvalue is always equal to one. Therefore, the MC process is always stable. However, considering thick layer thickness and short time steps, the sediment does not move to the adjacent state but always stays in current state. In this situation, the assumption that a particle is uniformly distributed in each layer will be violated. For the sake of this problem the relation between time steps and layer thickness should be determined. With available experimental data, using the Coefficient of determination r^2 (P.Krause et al. 2005), the model results with different time steps and layer thickness, see Figure 4 and Figure 5. The figure 4 (condition SF2) with higher Rouse number has high coefficient of determination under some conditions. Compared with different layer thickness, a model with layer thickness which is equal to 0.01 has best behavior in three conditions. However, Figure4 shows that r^2 suddenly decreases when the time step is much smaller in all three conditions. This constraint between time step and layer thickness is not a linear function. In lower Rouse number, the time step is recommended to be 0.1~1s. For a higher Rouse number, Figure 5 shows the comparison between conditions SF2 with the proposed model. Under suspended load dominant condition, the same layer thickness is not suitable in the case. Finer layer thickness is needed for lighter particle and faster flow condition.

CONCLUSION

The Motion of sediment particles can be seen as a stochastic process. The Multiple-state Discrete-time MC Approach is introduced in this study. In the two conditions (high and low Rouse number), the Multiple-state Discrete-time MC Approach provides good accuracy compared to experimental data (Wang and Qian 1989). r^2 is equal to 0.98 in the high Rouse number and equal to 0.93 in the lower Rouse number. For the stability, the Multiple-state Discrete-time MC approach is always stable but constrained by assumption Time steps need to be determined. In the study, the time step is implicitly decided. Condition under lower rouse number, layer thickness should be finer. The ratio between time steps and layer thickness is a nonlinear function (constant) which depends on the particle and flow condition.

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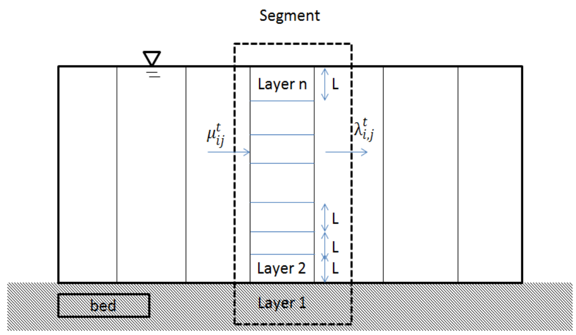

Fig. 1 The conception of the proposed model

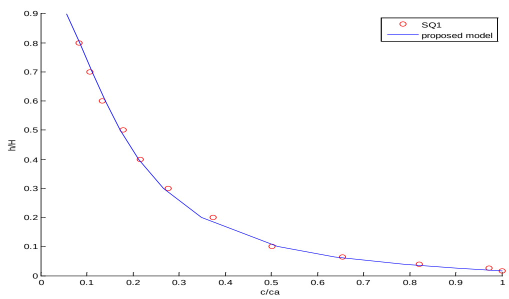

Fig. 2 Comparison between experiment data (SQ1) and propose mode with time step=0.3s layer thickness=0.05H. Rouse number=0.533

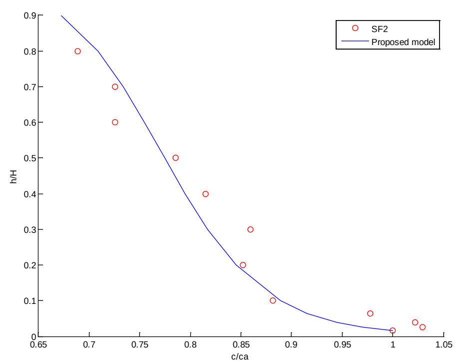

Fig. 3 Comparison between experiment data (SF2) and proposed model with time step=0.025s layer thickness=0.0025H. Rouse number=0.064

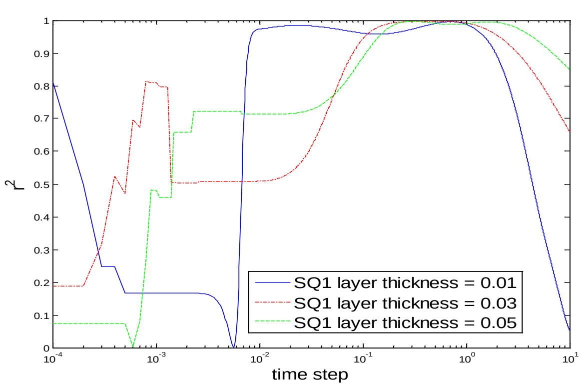

Fig. 4 The numerical analysis between the proposed model and experiment data(SQ1)

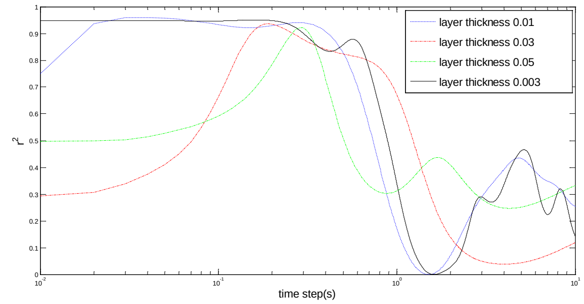

Fig. 5 The numerical analysis between the proposed model and experiment data(SF2)

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