Probabilistic Risk Assessment of Reservoir Operation

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ABSTRACT

The objective of this study was to provide a probabilistic risk assessment of the reservoir operation for the Shihmen Reservoir. The probability-based assessment was achieved by coupling a deterministic network flow reservoir operation model and a stochastic reservoir inflow simulation model. Stochastic inflow simulation was composed of a Poisson model for typhoon occurrences, a bivariate-gamma typhoon flood flow simulation model, and an ARMA(1,1) model for non-typhoon daily inflow simulations. Yearly series of simulated 10-day-period reservoir inflows were used in reservoir operation simulation using the network flow model. Results of reservoir operation simulation were then used for reservoir operation evaluation. It was found that the risk of agricultural water shortage was highest in February and lowest in September. The simulation results can also facilitate a drought frequency analysis.

KEY WORDS: Reservoir Operations; Network Flow Model; Flow Simulation

INTRODUCTION

From time to time, Taiwan experienced droughts which resulted in various degrees of water shortage in different seasons and different regions. Although a sound reservoir operation practice can alleviate the economic losses of water shortages, effects of reservoir operation are highly variable due to the random nature of reservoir inflow. Reservoir operations in Taiwan are based on rule curves which generally provide satisfactory results. However, there have been concerns about the performance of reservoir operations based on rule curves, and quantitative evaluations of reservoir operations have been conducted. For instance, Huang et al. (2000) used simulation methods to evaluate the performance of the rule curves of the Shihmen reservoir. Using a minimum cost network flow model, Chou and Wu (2014) established a model, WRASIM, for reservoir operation simulation. The WRASIM model has since then been extensively used in reservoir operation simulation in Taiwan. In general, performance of the rule curves was often evaluated by simulating reservoir operations with the observed (deterministic) reservoir inflows series.

It is also desired to understand the risks of water shortage in different seasons and the probability of occurrences of hydrological droughts (Wilhite & Glantz, 1985) through a quantitative evaluation of the performance of reservoir operations. A critical issue in drought frequency analysis is that the sample size of observed droughts may be too small for a reliable estimation. In this study, a large number of simulated reservoir inflow series were used to circumvent the problem of sample size, and a network flow model was used to simulate the reservoir operation.

Besides the network flow model, other simulation and optimization models have also been used in simulating reservoir planning and management. The network flow model was chosen for this study because of (1) its capability of providing an upper bound of performance that can be achieved in reality (Yerramddy and Wurbs, 1996); (2) its intuitive and tractable decision making framework; and (3) its superior computing efficiency among the optimization models.

METHODS

Synthetic Flows

Typhoons mostly occur during the summer season (July – October) and flood flows induced by typhoons are much higher in magnitudes and variations than non-typhoon flows. Therefore, in this study, daily typhoon and non-typhoon flows were independently generated and then combined to form the synthetic daily flow series. Daily flows induced by individual typhoons were extracted from historical flow records by referencing to the warning periods of individual typhoons issued by the Central Weather Bureau and a lower threshold (185 m³/s-day) of daily flows. Following the typhoon-induced flows, a period of flow recession which characterizes withdrawal of stored water was also considered for generation of typhoon flood flows. After removal of typhoon flows, the observed daily non-typhoon flow series were interpolated by a nonparametric method to fill in missing non-typhoon flows.

Typhoon daily flows. Daily inflows of the Shihmen Reservoir of the 1975-2013 period were analyzed. There were 101 typhoon occurrences in the 39-year period. Flood flows induced by these typhoons were characterized by four characteristic variables – (1) inter-event time, (2) event-total flow, (3) duration and (4) annual counts of typhoons. As shown in figure 1, the inter-event time represents the time span between two typhoons; duration represents the elapsed time length of a typhoon; the event-total flow is the total flow volume of a typhoon. Table 1 summarizes the probability distributions of these typhoon characteristic variables through the chi-square goodness-of-fit tests.
Fig. 1 Characteristic variables of typhoons.

Table 1. Probability distributions of typhoon characteristic variables determined by the chi-square goodness-of-fit tests.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counts of typhoons</td>
<td>Poisson</td>
<td>0.224</td>
</tr>
<tr>
<td>Duration</td>
<td>Gamma</td>
<td>0.227</td>
</tr>
<tr>
<td>Event-total flow</td>
<td>Gamma</td>
<td>0.220</td>
</tr>
<tr>
<td>Inter-event time</td>
<td>Gamma</td>
<td>0.084</td>
</tr>
</tbody>
</table>

Non-typhoon daily flows. Non-typhoon daily flows exhibited periodic (seasonal) non-stationarity and a non-Gaussian distribution. Thus, the non-typhoon daily flow series \( Y_t \) was log-transformed and the resultant series \( \hat{Y}_t = \log(Y_t) \) were checked for presence of periodic components by examining its periodogram. Upon identifying significant periodic components \( 1/w_i, i = 1, K, n \), the log-transformed series \( \hat{Y}_t \) can be represented as follows:

\[
Y_t = \mu + \sum_{i=1}^{n} A_i \cos(w_i t) + B_i \sin(w_i t) + \epsilon_t
\]

where \( A_i \) and \( B_i \) are coefficients and \( w_i \) is the frequency. The periodogram (figure 2) of \( \hat{Y}_t \) suggests the existence of several periodic components, with periods of 6.7 to 8 years and one single year. These periodic components capture the seasonal variation pattern of the non-typhoon flows.

\[
\hat{Y}_t = 3.32 + 0.29\cos(0.0003t) - 0.4\sin(0.0003t) - 0.39\cos(0.0004t) + 0.23\sin(0.0004t) + 0.12\cos(0.00027t) - 0.1\sin(0.00027t)
\]

After removing the seasonal effect, the flow was fitted by an ARMA(1,1), according to the smaller AIC and the principal of parsimonious. The fitted ARMA models of various parameter settings were listed in Table 2 and Table 3.

Table 2 ARMA model fitting for non-typhoon flows.

<table>
<thead>
<tr>
<th>ARMA(p,q)</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(1,0)</td>
<td>5752.53</td>
</tr>
<tr>
<td>ARMA(0,1)</td>
<td>21240.81</td>
</tr>
<tr>
<td>ARMA(1,1)</td>
<td>5691.55</td>
</tr>
<tr>
<td>ARMA(2,1)</td>
<td>5555.12</td>
</tr>
<tr>
<td>ARMA(1,2)</td>
<td>5618.17</td>
</tr>
<tr>
<td>ARMA(2,2)</td>
<td>5555.80</td>
</tr>
</tbody>
</table>

Table 3 Coefficients of ARMA(1,1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimation</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0</td>
<td>0.0372</td>
</tr>
<tr>
<td>AR1</td>
<td>0.9387</td>
<td>0.0031</td>
</tr>
<tr>
<td>MA1</td>
<td>-0.0777</td>
<td>0.0098</td>
</tr>
</tbody>
</table>

Synthetic flows simulation. Non-typhoon daily flows were generated by coupling the ARMA(1,1) simulation with parameters listed in Table 3 and periodic flows simulation using equation (2). Simulation of typhoon-induced flood flows was achieved by conducting Poisson simulation of annual number of typhoon occurrence, followed by a bivariate gamma simulation of the typhoon duration and event-total flows. All parameters were estimated by the maximum likelihood method. The duration and the event-total flow of typhoon events were highly correlated with a correlation coefficient of 0.879. The bivariate gamma distribution has been widely used in hydrology (Yue et al., 2001). In this study, the typhoon duration and event-total flows were simulated using a frequency-factor based approach of bivariate gamma simulation which was derived by Cheng et al. (2010).

Network Flow Model

The Shihmen Reservoir provides water supplies for different sectors including irrigation, industrial, and domestic and public usages on a ten-day-period (TDP) basis. The amounts of water demands vary with sectors and seasons in a year. Predetermined priorities have been assigned for different sectoral water supplies. In our network flow modeling, a set of cost coefficients reflecting such predetermined priorities was used. In the network flow model, each arc \( (x_{ij}) \) represents a particular water supply. Most reservoir system analyses were formulated as a minimum cost flow problem, which can be presented in the following format:

Minimize:

\[
\sum_i \sum_j c_{ij}x_{ij}
\]

Subject to:

\[
\sum_j x_{ij} - \sum_j x_{ji} = 0
\]

\[
l_{ij} \leq x_{ij} \leq u_{ij}
\]
where \( i = 1, \ldots, m \) and \( j = 1, \ldots, n \); \( m = n \) = the numbers of nodes; \( x_{ij} \) is an arc, meaning the quantity flowing from node \( i \) to node \( j \); \( c_{ij} \) is the unit cost of flow from node \( i \) to \( j \); \( l_{ij} \) and \( u_{ij} \) is the lower bound and the upper bound of the flow that allowed in arc \( x_{ij} \). Because the network flow model aimed to minimize the total cost, arcs of higher priorities were assigned lower cost coefficients, and the model would therefore supply water to individual arcs based on their priorities and achieve the minimum cost for the whole reservoir water supply system.

**Priority order.** The flows were distributed into three target releases - the agricultural release (RA\(_{t}\)), the public release (RP\(_{t}\)) which represents the total release for domestic and industrial usages, and the storage (S\(_{t}\)) in the period \( t \). However, with a set of three rule curves, any of the three target releases was divided into four components:

\[
\sum \sum \sum RA_{t}^{*} = \sum \sum \sum RP_{t}^{*} = \sum \sum \sum S_{t}^{*}
\]

\( \sum \) where the subscript \( i \) denotes the status of the water stage. The relation of the status and the rule curves was shown in figure 3.

![Fig. 3 Rule curves of the Shihmen Reservoir and illustration of the four release components.](image)

Priorities of these 12 target release components should be determined by considering the water right, operating rule curves, and the physical law. The superscript * of the notation \( X^{*} \) represents the priority of the target release (\( X \)). Management of the Shihmen Reservoir requires the agricultural usage be given a higher priority, and thus the following priority order was given in this study.

\[
RA_{t,i}^{*} > RP_{t,i}^{*} > S_{t,i}^{*} \quad i = 1,2,3
\]

(7)

The reservoir operation further requires that, in case of reservoir storage being lower than the rule curve, demands were supplied at discounted rates. The following priority order was given

\[
RA_{t,i}^{*} > RP_{t,i}^{*} > S_{t,i}^{*} \quad i = 1,2,3
\]

(8)

Physical law requires that the status close to the bottom should be satisfied first. And, thus the priority order was

\[
RA_{t+1,i}^{*} > RP_{t+1,i}^{*} > RA_{t,i}^{*} > S_{t,i}^{*} \quad i = 1,2,3
\]

(9)

\[
RP_{t+1,i}^{*} > S_{t+1,i}^{*} > RP_{t,i}^{*} > S_{t,i}^{*} \quad i = 1,2,3
\]

(10)

**RESULTS**

**Synthetic flows**

Synthetic flows generated in this study were evaluated by comparing the statistical characteristic of the historical and synthetic flows. Table 4 shows that the interval of the 2.5 percent quantile and 97.5 percent quantile of the simulation results included the history record. Thus, synthetic flows were close to the reality.

<table>
<thead>
<tr>
<th>Variable</th>
<th>2.5% quantile</th>
<th>97.5% quantile</th>
<th>History</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of Duration (m³/s-day)</td>
<td>3.34</td>
<td>4.01</td>
<td>3.01</td>
</tr>
<tr>
<td>S.E. of Duration (m³/s-day)</td>
<td>1.36</td>
<td>1.98</td>
<td>1.70</td>
</tr>
<tr>
<td>Mean of event-total flow (m³/s-day)</td>
<td>1652.31</td>
<td>2349.79</td>
<td>1843.30</td>
</tr>
<tr>
<td>S.E. of event-total flow (m³/s-day)</td>
<td>1309.21</td>
<td>2191.76</td>
<td>1743.68</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.73</td>
<td>0.89</td>
<td>0.84</td>
</tr>
<tr>
<td>Counts of typhoons</td>
<td>1.92</td>
<td>2.77</td>
<td>2.59</td>
</tr>
<tr>
<td>Mean of flow (m³/s-day)</td>
<td>44.71</td>
<td>54.53</td>
<td>47.79</td>
</tr>
<tr>
<td>S.E. of flow (m³/s-day)</td>
<td>91.96</td>
<td>126.66</td>
<td>130.57</td>
</tr>
</tbody>
</table>

**Network flow model**

Performance of the network flow model was also evaluated by comparing its simulation results against historical record of reservoir operation. Figure 4 demonstrates that the results of the network flow model simulation (blue line) and the historical operation record (red line) had a similar pattern, suggesting that reasonable cost coefficients were set in the network flow model and it could mimic the real reservoir operation.
Risk assessment

From the results of network flow simulation, continuous periods (or durations) of water shortage and the corresponding cumulative amounts of water shortage could be determined. In this study, hydrological droughts were defined as water-shortage events having durations longer than two ten-day-periods and the cumulative amount of water shortage was defined as the severity of a drought event. Figure 5 illustrates the relation between the duration and the severity of drought events. The large number of simulated drought events, with various durations and levels of severity, can be used to estimate the return period of a specific drought event.

For a given TDP, the probability distribution of water shortage rate can be derived. Water shortage rate of 0.3 is considered as a critical threshold for occurrences of agricultural droughts because crop yields may be affected if the shortage rate exceeds 0.3. Figure 6 illustrates the exceedance probabilities of the critical water shortage rate of individual TDPs. The exceedance probability represents the risk of critical agricultural water shortage. It was found that the risk of agricultural water shortage was highest in February and lowest in September. Such results are consistent with the reality in the study area that fallow often occurred in February due to larger demands of irrigation water supplies in the dry season.

CONCLUSIONS

This study provides a probabilistic risk assessment of the Shihmen reservoir operation based on the network flow model and synthetic flows. By a suitable formulation and an appropriate setting of cost coefficients, the network flow model could simulate the reservoir operation well. Synthetic flows were composed of the non-typhoon daily flows, fitted by a log-normal ARMA(1,1) model and typhoon daily flows, in which occurrences of the typhoon were modeled by a Poisson process and their durations and event-total flows were characterized by a bivariate gamma distribution. A few concluding remarks were drawn as follows:

1. Not only can the network flow model simulate the reservoir operation of the Shihmen Reservoir well, it also provide a benchmark of reservoir operation.
2. Separation of the flood flows induced by typhoons and non-typhoon flows is beneficial to generation of synthetic daily flows.
3. The duration and event-total flow of typhoon-induced floods can be characterized by a bivariate gamma distribution.
4. The risk of agricultural water shortage was highest in February and lowest in September in our study area.

REFERENCES